KOSHI TEN`SIZLIGINEN PAYDALANIP TEN`SIZLIKLERDI DA`LILLEW USILLARI

Djumabaev N.
A`jiniyaz atindag`i No`kis Ma`mleketlik pedagogikaliq institutinin` «Matematikani oqitiw metodikasi» kafedrasinin` ag`a oqitiwshisi, pedagogika ilimlerinin` kandidati

Tayanch so'zlar: далиллаш усуллари, Коши тенгсизлиги, математика, фойдаланиш,элементар математика.
Ключевые слова: методы доказательства, неравенство Коши, математика, применение, элементарная математика.
Key words: proof methods, Cauchy inequality, mathematics, application, elementary mathematics.

РЕЗЮМЕ:
Ушбу мақолада Коши тенгсизлигидан фойдаланиб тенгсизликни далиллаш усуллари берилган.
РЕЗЮМЕ:
В этой статье представлены методы доказательства неравенств с использованием неравенства Коши.
SUMMARY:
This paper presents methods for proving inequalities using Cauchy's inequality.

Ten`sizliklerdi da`lillewde Koshi ten`sizligi u`lken a`hmiyetke iye. Koshi ten'sizliginin' uliwma tu'rin to'mendegishe jaziwg'a boladi:

Eger $a_1 \geq 0, a_2 \geq 0, ... a_n \geq 0$ bolsa,

$$\frac{a_1 + a_2 + ... + a_n}{n} \geq \sqrt[n]{a_1 a_2 ... a_n} \qquad (1)$$

bunda $n \geq 2$.Bul ten'sizliktegi ten'lik $a_1 = a_2 = ... a_n$ bolg'anda orinli boladi.

Dara jag'dayda (1) ten'sizlik n=2 bolg'anda to'mendegishe boladi:

$$\frac{a_1 + a_2}{2} \geq \sqrt{a_1 a_2} \qquad (2)$$

Eger (2) formulada $a_1 = a$ ha'm $a_2 = \frac{1}{a}$, bunda $a > 0$ bolsa, onda

$$a + \frac{1}{a} \geq 2 \qquad (3)$$

tu'rinde boladi, bunda ten'lik a=1 bolg'anda orinli boladi.

(3) ten'sizlik **a** nin' teris ma'nisinde,yag'niy $a<0$ bolg'anda

$$a+\frac{1}{a}\le -2$$

tu'rinde boladi,bunda ten'lik a=-1 bolg'anda orinli boladi.

Bul keltirilgen Koshi ten'sizliginen paydalanip ten'sizliklerdi da'lillewdi qarap o'teyik:

1-misal: Eger $a\ge 0$ ha'm $b\ge 0$ bolsa $(a+2)(b+2)(a+b)\ge 16ab$ ten'sizligin da'lillen'.

Da'lillew: Berilgen ten'sizliktegi ha'r bir ko'beyiwshige Koshi ten'sizligin paydalanip to'mendegishe jazamiz:

$$a+2\ge 2\sqrt{2a},\quad b+2\ge 2\sqrt{2b},\quad a+b\ge 2\sqrt{ab}$$

Payda bolg'an bul ten'sizliklerdi o'z-ara ko'beytemiz:

$$(a+2)(b+2)(a+b)\ge 2\sqrt{2a}\cdot 2\sqrt{2b}\cdot 2\sqrt{ab}=16ab$$

Berilgen ten'sizlik da'lillendi.

2-misal: $3\left(a^3+b^3+c^3\right)\ge (a+b+c)(ab+ac+bc)$ ten'sizligin da'lillen'.

Da'lillew: (1) tu'rindegi Koshi ten'sizliginin' n=3 bolg'an jag'dayinan paydalanip izbe-iz tu'rde to'mendegi ten'sizliklerdi payda etemiz:

$$a^3+b^3+c^3\ge 3abc$$
$$a^3+b^3+c^3\ge 3abc$$
$$a^3+b^3+c^3\ge 3abc$$
$$a^3+a^3+b^3\ge 3a^2b$$
$$a^3+b^3+b^3\ge 3ab^2$$
$$a^3+a^3+c^3\ge 3a^2c$$
$$a^3+c^3+c^3\ge 3ac^2$$
$$b^3+b^3+c^3\ge 3b^2c$$
$$b^3+c^3+c^3\ge 3bc^2$$

Bul ten'sizliklerdin' sol jag'in sol jag'ina ha'm on' jag'in on' jag'ina qosip to'mendegi ten'sizliklerdi payda etemiz:

$$9\left(a^3+b^3+c^3\right)\ge 3\left(3abc+a^2b+ab^2+a^2c+ac^2+b^2c+bc^2\right)\Leftrightarrow$$
$$\Leftrightarrow 3\left(a^3+b^3+c^3\right)\ge 3abc+a^2b+ab^2+a^2c+ac^2+b^2c+bc^2=abc+a^2b+ab^2+abc+a^2c+ac^2+$$
$$+abc+b^2c+bc^2=ab(a+b+c)+ac(a+b+c)+bc(a+b+c)=(a+b+c)(ab+ac+bc)$$

Berilgen ten'sizlik da'lilllendi.

3-misal: $abcd\le \dfrac{a^5+b^5+c^5+d^5}{a+b+c+d}$ ten'sizligin da'lillen',

bunda $a\ge 0,\ b\ge 0,\ c\ge 0,\ d\ge 0, a+b+c+d>0$.

Da'lillew: $a\ge 0,\ b\ge 0,\ c\ge 0,\ d\ge 0$ bolg'anliqtan (1) tu'rindegi Koshi ten'sizliginin' n=5 bolg'an jag'dayinan paydalanip to'mendegi ten'sizliklerdi jazamiz:

$$a^5+a^5+b^5+c^5+d^5\ge 5\sqrt[5]{a^5\cdot a^5\cdot b^5\cdot c^5\cdot d^5}=5a^2bcd$$
$$a^5+b^5+b^5+c^5+d^5\ge 5\sqrt[5]{a^5\cdot b^5\cdot b^5\cdot c^5\cdot d^5}=5ab^2cd$$
$$a^5+b^5+c^5+c^5+d^5\ge 5\sqrt[5]{a^5\cdot b^5\cdot c^5\cdot c^5\cdot d^5}=5abc^2d$$

$$a^5+b^5+c^5+d^5+d^5\ge 5\sqrt[5]{a^5\cdot b^5\cdot c^5\cdot d^5\cdot d^5}=5abcd^2$$

Bul ten'sizliklerdin' sol jag'in sol jag'ina ha'm on' jag'in on' jag'ina qosip to'mendegi ten'sizlikti payda etemiz:

$$5\left(a^5+b^5+c^5+d^5\right)\ge 5\left(a^2bcd+ab^2cd+abc^2d+abcd^2\right)\Leftrightarrow$$
$$\Leftrightarrow a^5+b^5+c^5+d^5\ge abcd\left(a+b+c+d\right)$$

Sha'rt boyinsha a+b+c+d>0 bolg'anliqtan, $a^5+b^5+c^5+d^5\ge abcd$ ten'sizligi kelip shig'adi.

4-misal: Eger $a^2+b^2+c^2\le 3$ ha'm $a\ne 0, b\ne 0, c\ne 0$ bolsa

$$\frac{a^2}{1+a^4}+\frac{b^2}{1+b^4}+\frac{c^2}{1+c^4}\le\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}$$ ten'sizligin da'lillen'.

Da'lillew: Berilgen ten'sizliktin' sol jag'in (3) tu'rindegi ten'sizlikten paydalanip to'mendegi ko'riniste jazip alamiz

$$\frac{a^2}{1+a^4}+\frac{b^2}{1+b^4}+\frac{c^2}{1+c^4}=\frac{1}{a^2+\frac{1}{a^2}}+\frac{1}{b^2+\frac{1}{b^2}}+\frac{1}{c^2+\frac{1}{c^2}}\le\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$$

Al, on' jag'in Koshi ten'sizligi boyinsha

$$\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}\ge\frac{3}{\sqrt[3]{\left(1+a^2\right)\left(1+b^2\right)\left(1+c^2\right)}}\ge\frac{9}{3+a^2+b^2+c^2}$$

$a^2+b^2+c^2\le 3$ sha'rtine tiykarlanip keyingi ten'sizlikti to'mendegishe jazamiz:

$$\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}\ge\frac{9}{3+a^2+b^2+c^2}\ge\frac{9}{3+3}=\frac{3}{2}$$

Demek berilgen ten'sizliktin' sol jag'i $\frac{3}{2}$ kishi ya'ki ten', al on' jag'i $\frac{3}{2}$ ten u'lken ya'ki ten', olay bolsa berilgen ten'sizlik da'lillendi.

5-misal: Eger a>0, b>0, c>0 bolsa

$$\left(\frac{a+b}{c}\right)^n+\left(\frac{b+c}{a}\right)^n+\left(\frac{c+a}{b}\right)^n\ge 3\cdot 2^n$$ ten'sizligin da'lillen'.

Da'lillew: (1) tu'rindegi Koshi ten'sizliginin' n=3 bolg'an jag'dayinan paydalanip ten'sizlikti to'mendegishe jazamiz:

$$\left(\frac{a+b}{c}\right)^n+\left(\frac{b+c}{a}\right)^n+\left(\frac{c+a}{b}\right)^n\ge 3\cdot\sqrt[3]{\left(\frac{a+b}{c}\right)^n+\left(\frac{b+c}{a}\right)^n+\left(\frac{c+a}{b}\right)^n}=$$
$$=3\cdot\left(\sqrt[3]{\frac{(a+b)\cdot(b+c)\cdot(c+a)}{abc}}\right)^n\ge 3\left(\sqrt[3]{\frac{2\sqrt{ab}\cdot 2\sqrt{bc}\cdot 2\sqrt{ca}}{abc}}\right)^n=3\left(\sqrt[3]{\frac{8abc}{abc}}\right)=3\cdot 2^n$$

Demek, berilgen ten'sizlik da'lillendi.

6-misal: $a>0$ ha'm $b>0$ bolsa,

$$\frac{1}{\sqrt[4]{a}}+\frac{1}{\sqrt[4]{b}}\ge\frac{2\sqrt[4]{2}}{\sqrt[4]{a+b}}$$ ten'sizligin da'lillen'.

Da'lillew: Koshi ten'sizliginin' (2) formulasinan paydalanip ten'sizlikti to'mendegishe jazamiz:

$a+b\ge 2\sqrt{ab}$ ten'sizliginin' eki jag'inan to'rtinshi da'rejeli koren alamiz:

$$\sqrt[4]{a+b}\ge\sqrt[4]{2}\cdot\sqrt[8]{ab}\Rightarrow\frac{1}{\sqrt[8]{ab}}\ge\frac{\sqrt[4]{2}}{\sqrt[4]{a+b}}$$

Berilgen ten'sizliktin' sol jag'in Koshi ten'sizliginin' (3) formulasinan paydalanip to'mendegishe jazamiz:

$$\frac{1}{\sqrt[4]{a}}+\frac{1}{\sqrt[4]{b}}\geq\frac{2}{\sqrt[8]{ab}}$$

bunnan, $\frac{1}{\sqrt[4]{a}}+\frac{1}{\sqrt[4]{b}}\geq\frac{2}{\sqrt[8]{ab}}\geq\frac{2\sqrt[4]{2}}{\sqrt[4]{a+b}}$

Demek, berilgen ten'sizlik da'lillendi.

A'debiyatlar:

1. I.X.Sivashinskiy. Neravenstva v zadachax. Izdatelstvo "Nawka". Moskva.1967.
2. V.N.Litvinenko, A.G.Mordkovich. Praktikum po elementarnoy matematike. Moskva „Prosveshenie" 1991.
3. T.To'laganov, A.Normatov. Matematikadan praktikum. Toshkent, «O'qituvchi», 1989.
4. http://w.w.w.ziyonet.uz/